Ecological Economics 101 (2014) 90–102

Contents lists available at ScienceDirect

Ecological Economics

journal homepage: www.elsevier.com/locate/ecolecon

Methodological and Ideological Options

Human and nature dynamics (HANDY): Modeling inequality and use of resources in the collapse or sustainability of societies

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ARTICLE INFO

Article history:
Received 1 October 2012
Received in revised form 11 February 2014
Accepted 18 February 2014
Available online 2 April 2014

Keywords:
Human–nature dynamics
Societal collapse
Carrying capacity
Overshoot vs. sustainability
Economic inequality
Ecological strain

ABSTRACT

There are widespread concerns that current trends in resource-use are unsustainable, but possibilities of overshoot/collapse remain controversial. Collapses have occurred frequently in history, often followed by centuries of economic, intellectual, and population decline. Many different natural and social phenomena have been invoked to explain specific collapses, but a general explanation remains elusive.

In this paper, we build a human population dynamics model by adding accumulated wealth and economic inequality to a predator–prey model of humans and nature. The model structure, and simulated scenarios that offer significant implications, are explained. Four equations describe the evolution of Elites, Commoners, Nature, and Wealth. The model shows Economic Stratification or Ecological Strain can independently lead to collapse, in agreement with the historical record.

The measure "Carrying Capacity" is developed and its estimation is shown to be a practical means for early detection of a collapse. Mechanisms leading to two types of collapses are discussed. The new dynamics of this model can also reproduce the irreversible collapses found in history. Collapse can be avoided, and population can reach a steady state at maximum carrying capacity if the rate of depletion of nature is reduced to a sustainable level and if resources are distributed equitably.

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1. Introduction

There are widespread concerns that current trends in population and resource-use are unsustainable, but the possibilities of an overshoot and collapse remain unclear and controversial. How real is the possibility of a societal collapse? Can complex, advanced civilizations really collapse? It is common to portray human history as a relentless and inevitable trend toward greater levels of social complexity, political organization, and economic specialization, with the development of more complex and capable technologies supporting ever-growing population, all sustained by the mobilization of ever-increasing quantities of material, energy, and information. Yet this is not inevitable. In fact, cases where this seemingly near-universal, long-term trend has been severely disrupted by a precipitous collapse – often lasting centuries – have been quite common. A brief review of some examples of collapses suggests that the process of rise-and-collapse is actually a recurrent cycle found throughout history, making it important to establish a general explanation of this process (Chase-Dunn and Hall, 1997; Goldstein, 1988; Meadows et al., 1972; Modelski, 1987; Tainter, 1988; Turchin and Nefedov, 2009; Yoffee and Cowgill, 1988).

The Roman Empire's dramatic collapse (followed by many centuries of population decline, economic deterioration, intellectual regression, and the disappearance of literacy) is well known, but it was not the first rise-and-collapse cycle in Europe. Prior to the rise of Classical Greco-Roman civilization, both the Minoan and Mycenaean Civilizations had each risen, reached very advanced levels of civilization, and then collapsed virtually completely (Morris, 2006; Redman, 1999). The history of Mesopotamia – the very cradle of civilization, agriculture, complex society, and urban life – presents a series of rise-and-declines including the Sumerians, the Akkadian, Assyrian, Babylonian, Achaemenid, Seleucid, Parthian, Sassanid, Umayyad, and Abbasid Empires (Redman et al., 2004; Yoffee, 1979). In neighboring Egypt, this cycle also appeared repeatedly. In both Anatolia and in the Indus Valley, the very large and long-lasting Hittite and Harrapan civilizations both collapsed so completely that their very existence was unknown until modern archeology rediscovered them. Similar cycles of rise and collapse occurred repeatedly in India, most notably with the Mauryan and the Gupta Empires (Edwards et al., 1971, 1973; Jansen et al., 1991; Kenoyer, 1998; Thapar, 2004). Southeast Asia similarly experienced "multiple

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http://dx.doi.org/10.1016/j.ecolecon.2014.02.014
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and overlapping histories of collapse and regeneration" over 15 centuries, culminating in the Khmer Empire based in Angkor, which itself was depopulated and swallowed by the forest during the 15th Century (Stark, 2006). Chinese history is, very much like Egypt's, full of repeated cycles of rises and collapses, with each of the Zhou, Han, Tang, and Song Empires followed by a very serious collapse of political authority and socioeconomic progress (Chu and Lee, 1994; Lee, 1931; Needham and Wang, 1956).

Collapses are not restricted to the "Old World". The collapse of Maya Civilization is well known and evokes widespread fascination, both because of the advanced nature of Mayan society and because of the depth of the collapse (Demerest et al., 2004; Webster, 2002). As Diamond (2005) puts it, it is difficult to ignore "the disappearance of between 90 and 99% of the Maya population after A.D. 800 …and the disappearance of kings, Long Count calendars, and other complex political and cultural institutions." In the nearby central highlands of Mexico, a number of powerful states also rose to high levels of power and prosperity and then rapidly collapsed, Teotihuacan (the sixth largest city in the world in the 7th C) and Monte Alban being just the largest of these to experience dramatic collapse, with their populations declining to about 20–25% of their peak within just a few generations (Tainter, 1988).

We know of many other collapses including Mississippian Cultures such as Cahokia, South West US cultures such as the Pueblo and Hohokam, Andean civilizations such as Tiwanaku, Sub-Saharan civilizations such as Great Zimbabwe, and many collapses across the Pacific Islands, such as Easter Island. It is also likely other collapses have also occurred in societies that were not at a sufficient level of complexity to produce written records or archeological evidence. Indeed, a recent study (Shennan et al., 2013) of the Neolithic period in Europe has shown that "in contrast to the steady population growth usually assumed, the introduction of agriculture into Europe was followed by a boom-and-bust pattern in the density of regional populations". Furthermore "most regions show more than one boom–bust pattern", and in most regions, population declines "of the order of the 30–60%" can be found. The authors also argue that, rather than climate change or diseases, the timing and evidence point to endogenous causes for these collapses in 19 out of 23 cases studied, suggesting the possibility of "rapid population growth driven by farming to unsustainable levels". Moreover, through wavelet analysis of the archeological data, S. Downey [personal communication] has shown that the average length of such boom-and-bust cycles is about 300–500 years.

In summary, despite the common impression that societal collapse is rare, or even largely fictional, "the picture that emerges is of a process recurrent in history, and global in its distribution" (Tainter, 1988). See also Yoffee and Cowgill (1988), Goldstein (1988), Ibn Khaldun (1958), Kondratieff (1984), and Parsons (1991). As Turchin and Nefedov (2009) contend, there is a great deal of support for "the hypothesis that secular cycles — demographic–social–political oscillations of a very long period (centuries long) are the rule, rather than an exception in the large agrarian states and empires."

This brings up the question of whether modern civilization is similarly susceptible. It may seem reasonable to believe that modern civilization, armed with its greater technological capacity, scientific knowledge, and energy resources, will be able to survive and endure whatever crises historical societies succumbed to. But the brief overview of collapses demonstrates not only the ubiquity of the phenomenon, but also the extent to which advanced, complex, and powerful societies are susceptible to collapse. The fall of the Roman Empire, and the equally (if not more) advanced Han, Mauryan, and Gupta Empires, as well as so many advanced Mesopotamian Empires, are all testimony to the fact that advanced, sophisticated, complex, and creative civilizations can be both fragile and impermanent.

A large number of explanations have been proposed for each specific case of collapse, including one or more of the following: volcanoes, earthquakes, droughts, floods, changes in the courses of rivers, soil degradation (erosion, exhaustion, salinization, etc.), deforestation, climate change, tribal migrations, foreign invasions, changes in technology (such as the introduction of ironworking), changes in the methods or weapons of warfare (such as the introduction of horse cavalry, armored infantry, or long swords), changes in trade patterns, depletion of particular mineral resources (e.g., silver mines), cultural decline and social decadence, popular uprisings, and civil wars. However, these explanations are specific to each particular case of collapse rather than general. Moreover, even for the specific case where the explanation applies, the society in question usually had already experienced the phenomenon identified as the cause without collapsing. For example, the Minoan society had repeatedly experienced earthquakes that destroyed palaces, and they simply rebuilt them more splendidly than before. Indeed, many societies experience droughts, floods, volcanoes, soil erosion, and deforestation with no major social disruption (Tainter, 1988).

The same applies to migrations, invasions, and civil wars. The Roman, Han, Assyrian, and Mauryan Empires were, for centuries, completely militarily hegemonic, successfully defeating the neighboring "barbarian" peoples who eventually did overrun them. So external military pressure alone hardly constitutes an explanation for their collapses. With both natural disasters and external threats, identifying a specific cause compels one to ask, "yes, but why did this particular instance of this factor produce the collapse?" Other processes must be involved, and, in fact, the political, economic, ecological, and technological conditions under which civilizations have collapsed have varied widely. Individual collapses may have involved an array of specific factors, with particular triggers, but a general explanation remains elusive. Individual explanations may seem appropriate in their particular case, but the very universal nature of the phenomenon implies a mechanism that is not specific to a particular time period of human history, nor a particular culture, technology, or natural disaster (Tainter, 1988; Turchin, 2003; Yoffee and Cowgill, 1988).

In this paper we attempt to model collapse mathematically in a more general way. We propose a simple model, not intended to describe actual individual cases, but rather to provide a general framework that allows carrying out "thought experiments" for the phenomenon of collapse and to test changes that would avoid it. This model (called HANDY, for Human and Nature DYnamics) advances beyond existing biological dynamic population models by simultaneously modeling two separate important features which seem to appear across so many societies that have collapsed: (1) the stretching of resources due to the strain placed on the ecological carrying capacity (Abel, 1980; Catton, 1980; Kammen, 1994; Ladurie, 1987; Ponting, 1991; Postan, 1966; Redman, 1999; Redman et al., 2004; Wood, 1998; Wright, 2004), and (2) the economic stratification of society into Elites and Masses (or "Commoners") (Brenner, 1985; Parsons, 1991; Turchin, 2005, 2006; Turchin and Nefedov, 2009; Diamond, 2005; Goldstone, 1991; Ibn Khaldun, 1958). In many of these historical cases, we have direct evidence of Ecological Strain and Economic Stratification playing a central role in the character or in the process of the collapse (Culbert, 1973; Diamond, 2005; Goldstone, 1991; Lentz, 2000; Mitchell, 1990). For these empirical reasons, and the theoretical ones explained in Section 3, our model incorporates both of these two features. Although similar to the Brander and Taylor (1998) model (hereafter referred to as "BT") in that HANDY is based on the classical predator–prey model, the inclusion of two societal classes introduces a much richer set of dynamical solutions, including cycles of societal and ecological collapse, as well as the possibility of smoothly reaching equilibrium (the ecological carrying capacity). We use Carrying Capacity in its biological definition: the population level that the resources of a particular environment can sustain over the long term (Catton, 1980; Cohen, 1995; Daly and Farley, 2003). In this paper, we call these environment resources "Nature".

The paper is organized as follows: Section 2 gives a brief review of the predator–prey model; Section 3 includes the mathematical description of HANDY; Section 4 covers a theoretical analysis of the model equilibrium and possible solutions; Section 5 presents examples of scenarios within three distinct types of societies; Section 6 gives an overall discussion of the scenarios from Section 5; and Section 7 offers a short summary of the paper and a discussion of future work.

2. Predator–Prey Model

The predator–prey model, the original inspiration behind HANDY, was derived independently by two mathematicians, Alfred Lotka and Vitto Volterra, in the early 20th century (Lotka, 1925; Volterra, 1926). This model describes the dynamics of competition between two species, say, wolves and rabbits. The governing system of equations is

$$\begin{cases} \dot{x} & = & (ay)x - bx \\ \dot{y} & = & cy - (dx)y \end{cases}. \tag{1}$$

In the above system, x represents the predator (wolf) population; y represents the prey (rabbit) population; a determines the predator's birth rate, i.e., the faster growth of wolf population due to availability of rabbits; b is the predator's death rate; c is the prey's birth rate; d determines the predation rate, i.e., the rate at which rabbits are hunted by wolves.

Rather than reaching a stable equilibrium, the predator and prey populations show periodic, out-of-phase variations about the equilibrium values

$$\begin{cases} x_e & = & c/d \\ y_e & = & b/a \end{cases}. \tag{2}$$

Note consistency of the units on the left and right hand sides of Eqs. (1) and (2). A typical solution of the predator–prey system can be seen in Fig. 1.

3. HANDY

As indicated above, Human And Nature DYnamics (HANDY) was originally built based on the predator–prey model. We can think of the human population as the "predator", while nature (the natural resources of the surrounding environment) can be taken as the "prey", depleted by humans. In animal models, carrying capacity is an upper ceiling on long-term population. When the population surpasses the carrying capacity, mechanisms such as starvation or migration bring the population back down. However, in the context of human societies,

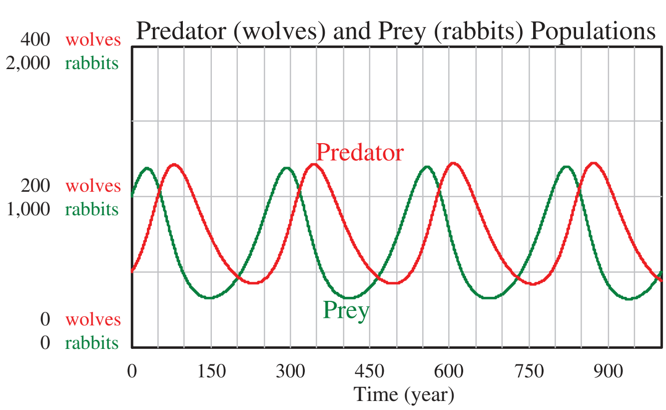


Fig. 1. A typical solution of the predator–prey system obtained by running the system with the following parameter values and initial conditions: $a = 3.0 \times 10^{-5}$ (rabbits·years)$^{-1}$; $b = 2.0 \times 10^{-2}$ years^{-1}, $c = 3.0 \times 10^{-2}$ years^{-1}, d $= 2.0 \times 10^{-4}$ (wolves·years)$^{-1}$; $x(0) = 1.0 \times 10^{+2}$ wolves; and $y(0) = 1.0 \times 10^{+3}$ rabbits. Predator population is measured in units of wolves, Prey population is measured in units of rabbits, and Time is measured in units of years.

the population does not necessarily begin to decline upon passing the threshold of carrying capacity, because, unlike animals, humans can accumulate large surpluses (i.e., wealth) and then draw down those resources when production can no longer meet the needs of consumption. This introduces a different kind of delay that allows for much more complex dynamics, fundamentally altering the behavior and output of the model. Thus, our model adds the element of accumulated surplus not required in animal models, but which we feel is necessary for human models. We call this accumulated surplus "wealth".

Empirically, however, this accumulated surplus is not evenly distributed throughout society, but rather has been controlled by an elite. The mass of the population, while producing the wealth, is only allocated a small portion of it by elites, usually at or just above subsistence levels. Based on this, and on the historical cases discussed in the introduction, we separated the population into "Elites" and "Commoners", and introduced a variable for accumulated wealth. For an analysis of this two-class structure of modern society, see Drăgulescu and Yakovenko (2001) and Banerjee and Yakovenko (2010). This adds a different dimension of predation whereby Elites "prey" on the production of wealth by Commoners. As a result, HANDY consists of four prediction equations: two for the two classes of population, Elites and Commoners, denoted by x_E and x_C, respectively; one for the natural resources or Nature, y; and one for the accumulated Wealth, w, referred to hereafter as "Wealth". This minimal set of four equations seems to capture essential features of the human–nature interaction and is capable of producing major potential scenarios of collapse or transition to steady state.

A similar model of population and renewable resource dynamics based on the predator–prey model was developed in the pioneering work of Brander and Taylor (1998) demonstrating that reasonable parameter values can produce cyclical "feast and famine" patterns of population and resources. Their model showed that a system with a slow-growing resource base will exhibit overshooting and collapse, whereas a more rapidly growing resource base will produce an adjustment of population and resources toward equilibrium values. They then applied this model to the historical case of Easter Island, finding that the model provides a plausible explanation of the population dynamics known about Easter Island from the archeological and scientific record. They thus argue that the Polynesian cases where population did collapse were due to smaller maximum resource bases (which they call "carrying capacity") that grew more slowly, whereas those cases which did not experience such a collapse were due to having a larger resource base (i.e., a larger carrying capacity). They then speculate that their model might be consistent with other historical cases of collapse, such as the ancient Mesopotamian and Maya civilizations or modern Rwanda.

However, the BT approach only models Population and Nature and does not include a central component of these historical cases: economic stratification and the accumulation of wealth. Thus, despite clear evidence for a stratified class structure in Easter Island's history prior to the collapse (as well as for Mesopotamia, the ancient Maya, and modern Rwanda), the BT model does not include class stratification as a factor. In their model, society produces and consumes as a single homogeneous unit. We feel that a historically realistic modeling of the evolution of human–nature dynamics in these stratified complex societies cannot be achieved without including this class stratification in the model. Brander and Taylor recognize that their model is simple, and that application to more complex scenarios may require further development of the structure of the model. We have found that including economic stratification, in the form of the introduction of Elites and Commoners, as well as accumulated Wealth, results in a much richer variety of solutions, which may have a wider application across different types of societies. HANDY's structure also allows for "irreversible" collapses, without the need to introduce an explicit critical depensation mechanism into the model as other models need to do. Thus while the Brander–Taylor model has only two equations, HANDY has four equations to predict the evolution of the rich and poor populations (Elites and Commoners), Nature, and accumulated Wealth (we examine other differences in Section 6.4 of the paper) The HANDY equations are given by:

$$\begin{cases} \dot{x}_C = \beta_C x_C - \alpha_C x_C \\ \dot{x}_E = \beta_E x_E - \alpha_E x_E \\ \dot{y} = \gamma y(\lambda - y) - \delta x_C y \\ \dot{w} = \delta x_C y - C_C - C_E. \end{cases} \quad (3)$$

It is to be noted that α_C, α_E, C_C, and C_E are all functions of w, x_C, and x_E. See Eqs. (4) and (6) and Fig. 2a and b.

3.1. Model Description

The total population is divided between the two variables, x_C and x_E, representing the population of commoners and of elites. The population grows through a birth rate β and decreases through a death rate α. β is assumed to be constant for both Elites and Commoners but α depends on Wealth as explained below.

In reality, natural resources exist in three forms: nonrenewable stocks (fossil fuels, mineral deposits, etc.), regenerating stocks (forests, soils, animal herds, wild fish stocks, game animals, aquifers, etc.), and renewable flows (wind, solar radiation, precipitation, rivers, etc.). Future generations of the model will disaggregate these forms. We have adopted a single formulation intended to represent an amalgamation of the three forms, allowing for a clear understanding of the role that natural resources play in collapse or sustainability of human societies.

Thus, the equation for Nature includes a regeneration term, $\gamma y(\lambda - y)$, and a depletion term, $-\delta x_C y$. The regeneration term has been written in the form of a logistic equation, with a regeneration factor, γ, exponential regrowth for low values of y, and saturation when y approaches λ, Nature's capacity — maximum size of Nature in absence of depletion. As a result, the maximum rate of regeneration takes place when $y = \lambda / 2$. Production is understood according to the standard Ecological Economics formulations as involving both inputs from, and outputs to, Nature (i.e., *depletion* of natural sources and *pollution* of natural sinks) (Daly, 1996; Daly and Farley, 2003). This first generation of HANDY models the *depletion* side of the equation as if it includes the reduction in Nature due to *pollution*.

The depletion term includes a rate of depletion per worker, δ, and is proportional to both Nature and the number of workers. However, the economic activity of Elites is modeled to represent executive, management, and supervisory functions, but not engagement in the direct extraction of resources, which is done by Commoners. Thus, only Commoners produce.

It is frequently claimed that *technological change* can reduce resource depletion and therefore increase carrying capacity. However, the effects of technological change on resource use are not unidirectional. Technological change can raise the efficiency of resource use, but it also tends to raise both per capita resource consumption and the scale of resource extraction, so that, absent policy effects, the increases in consumption often compensate for the increased efficiency of resource use. These are associated with the phenomena referred to as the Jevons Paradox, and the "Rebound Effect" (Greening et al., 2000; Polimeni et al., 2008; Ruth, 2009). For example, an increase in vehicle fuel efficiency tends to enable increased per capita vehicle miles driven, heavier cars, and higher average speeds, which then negate the gains from the increased fuel-efficiency. In addition, technological advances can enable greater resource extraction and throughput, which then appears as increases in the productivity of other factors of production. As Daly points out, much of the increase in productivity in both agriculture and industry in the last two centuries has actually come from increased (rather than decreased) resource throughput (Daly, 1991). A decline in the price of a resource is usually thought to reflect an increase in the abundance of that resource, but in fact, it often reflects that the resource is simply being extracted more rapidly. Rather than extend carrying capacity, this reduces it. Over the long-term, per capita resource-use has tended to rise over time despite dramatic technological advances in resource efficiency. Thus, the sign and magnitude of the effect of technological change on resource use varies and the overall effect is difficult to predict. Therefore, in this generation of HANDY, we assume that the effects of these trends cancel each other out. The model will be developed further to allow the rates of these technology-induced trends to be adjusted in either direction.

Finally, there is an equation for accumulated Wealth, which increases with production, $\delta x_C y$, and decreases with the consumption of the Elites and the Commoners, C_C and C_E, respectively. The consumption of the Commoners (as long as there is enough wealth to pay them) is $s x_C$, a subsistence salary per capita, s, multiplied by the working population. The Elites pay themselves a salary κ times larger, so that the consumption of the Elites is $\kappa s x_E$. However, when the wealth becomes too small to pay for this consumption, i.e., when $w < w_{th}$, the payment is reduced and eventually stopped, and famine takes place, with a much higher rate of death. κ is meant to represent here the factors that determine the division of the output of the total production of society between elites and masses, such as the balance of class power between elites and masses, and the capacity of each group to organize and pursue their economic interests. We recognize the inherent limitations, in this initial generation of our model, of holding that balance (κ) constant in each scenario, but we expect to develop κ further in later generations of HANDY so that it can be endogenously determined by other factors in the model.

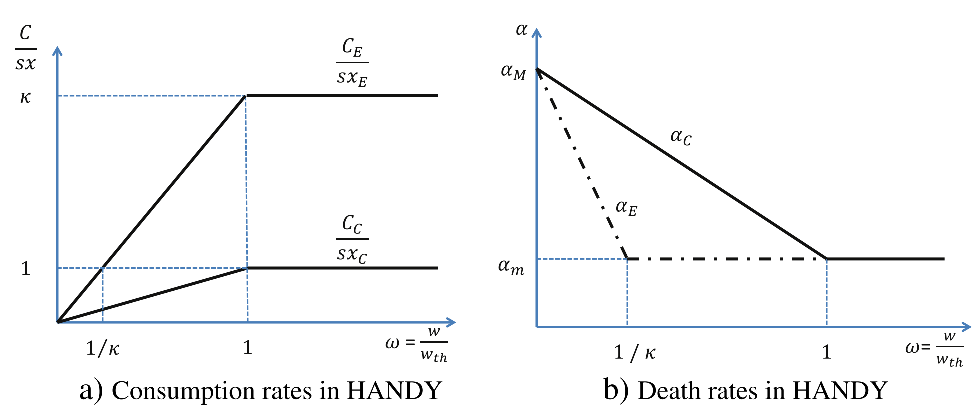


Fig. 2. Per capita Consumption rates and Death rates for Elites and Commoners as a function of Wealth. Famine starts when $\frac{C}{sx} \le 1$. Therefore, Commoners start experiencing famine when $\frac{w}{w_{th}} \le 1$, while Elites do not experience famine until $\frac{w}{w_{th}} \le \frac{1}{\kappa}$. This delay is due to Elites' unequal access to Wealth.

C_C and C_E, the consumption rates for the Commoner and the Elite respectively, are given by the following equations:

$$\begin{cases} C_C &= min\left(1, \frac{w}{w_{th}}\right) s x_C \\ C_E &= min\left(1, \frac{w}{w_{th}}\right) \kappa s x_E \end{cases}. \quad (4)$$

Wealth threshold, w_{th}, is a threshold value for wealth below which famine starts. It depends on the "minimum required consumption per capita", ρ:

$$w_{th} = \rho x_C + \kappa \rho x_E. \quad (5)$$

Even when Commoners start experiencing famine, i.e., when $w \leq w_{th}$, the Elites continue consuming unequally as indicated by the factor κ in the second term on the right hand side of Eq. (5). A graphical representation of the consumption rates are given in Fig. 2a.

The death rates for the Commoner and the Elite, α_C and α_E, are functions of consumption rates:

$$\begin{cases} \alpha_C &= \alpha_m + max\left(0, 1 - \frac{C_C}{s x_C}\right)(\alpha_M - \alpha_m) \\ \alpha_E &= \alpha_m + max\left(0, 1 - \frac{C_E}{s x_E}\right)(\alpha_M - \alpha_m) \end{cases}. \quad (6)$$

The death rates vary between a normal (healthy) value, α_m, observed when there is enough food for subsistence, and a maximum (famine) value, α_M that prevails when the accumulated wealth has been used up and the population starves. There are a variety of mechanisms which can reduce population when it exceeds carrying capacity, including everything from emigration, increased disease susceptibility, and outright starvation to breakdowns in social order and increased social violence, such as banditry, riots, rebellions, revolutions, and wars. These mechanisms are described in detail in Turchin (2003) but the net effect of all of them is a reduction in population, and that is what the dynamics of our model is meant to represent when we say "population decline" or "famine". Note also that an increase in the death rates (α) is equivalent to an equal decrease in the birth rates (β). The death rates α_C and α_E can be expressed in terms of $\frac{w}{w_{th}}$, a graphical representation of which is given Fig. 2b.

3.2. A Note on Units and Dimensions

There are three dimensions for quantities in HANDY:

1. Population (either Commoner or Elite), in units of *people*.
2. Nature/Wealth, in units of "*eco-Dollars*".
3. Time, in units of years.

The structure of the model requires Nature and Wealth to be measured with the same units, therefore we created the unit *eco-dollar*. Other parameters and functions in the model carry units that are compatible with the abovementioned dimensions following Eq. (3). For example, Carrying Capacity, χ, and the Maximum Carrying Capacity, χ_M, defined in Section 4.1, are both expressed in units of *people*.

4. Equilibrium Values and Carrying Capacity

We can use the model to find a sustainable equilibrium and maximum carrying capacity in different types of societies. In order for population to reach an equilibrium, we must have $\alpha_m \leq \beta_E \leq \beta_C \leq \alpha_M$. We define a dimensionless parameter, η:

$$\eta = \frac{\alpha_M - \beta_C}{\alpha_M - \alpha_m}. \quad (7)$$

Since we assume $\alpha_m \leq \beta_C \leq \alpha_M$, η will always be bounded by $0 \leq \eta \leq 1$.

4.1. Equilibrium when $x_E = 0$ (No Elites): Egalitarian Society

Assuming $x_E \equiv 0$, we can find the equilibrium values of the system (subscript "*e*" denotes the equilibrium values):

$$\begin{cases} x_{C,e} &= \frac{\gamma}{\delta}\left(\lambda - \eta \frac{s}{\delta}\right) \\ y_e &= \eta \frac{s}{\delta} \\ w_e &= \eta \rho x_{C,e} \end{cases}. \quad (8)$$

We define χ, the Carrying Capacity for the population, to be equal to $x_{C,e}$ in Eq. (8), i.e., the equilibrium value of the population in the absence of Elites:

$$\chi = \frac{\gamma}{\delta}\left(\lambda - \eta \frac{s}{\delta}\right). \quad (9)$$

Carrying Capacity can be maximized if Nature's regeneration rate is maximal, i.e., if $y_e = \frac{\lambda}{2}$. This requires δ to be set equal to a value δ_* that can result in a steady state with the maximum (sustainable) Population, which in this paper we call the "optimal" value of δ. From the second equation in Eq. (8), it can be seen that δ_* is given by:

$$\delta_* = \frac{2\eta s}{\lambda}. \quad (10)$$

The Maximum Carrying Capacity, χ_M, is thus given by:

$$\chi_M = \frac{\gamma}{\delta_*}\frac{\lambda}{2} = \frac{\gamma}{\eta s}\left(\frac{\lambda}{2}\right)^2. \quad (11)$$

4.2. Equilibrium when $x_E \geq 0$ and $\kappa = 1$ (No Inequality): Equitable Society

If we set $\kappa \equiv 1$ and $\beta_E \equiv \beta_C \equiv \beta$, we can reach an equilibrium state for which $x_E \geq 0$. This case models an equitable society of "Workers" and "Non-Workers". We need a dimensionless free parameter φ that sets the initial ratio of the Non-Workers to Workers:

$$\varphi = \frac{x_E(0)}{x_C(0)}. \quad (12)$$

The equilibrium values of the system can then be expressed as follows:

$$\begin{cases} x_{C,e} &= \frac{\gamma}{\delta}\left(\lambda - \eta \frac{s}{\delta}(1+\varphi)\right) \\ x_{E,e} &= \varphi x_{C,e} \\ y_e &= \eta \frac{s}{\delta}(1+\varphi) \\ w_e &= \eta \rho (1+\varphi) x_{C,e} \end{cases}. \quad (13)$$

The total population $x_e = x_{C,e} + x_{E,e}$ can still be maximized by choosing δ appropriately:

$$\delta_{**} = \frac{2\eta s}{\lambda}(1+\varphi). \quad (14)$$

This δ_{**} is larger than the optimal depletion factor given by Eq. (10). The difference arises because Workers have to produce more than they need just for themselves in order to support Non-Workers. For this choice of δ, total population is given by:

$$x_{e,M} = (1+\varphi)\frac{\gamma}{\delta_{**}}\frac{\lambda}{2} = \frac{\gamma}{\eta s}\left(\frac{\lambda}{2}\right)^2. \quad (15)$$

As can be seen from Eq. (15), maximum total population in equilibrium is independent of φ and conforms to the maximum carrying capacity given above by Eq. (11).

4.3. Equilibrium when $x_E \geq 0$ and $\kappa > 1$: Unequal Society

It is possible to attain equilibrium in an unequal society if we can satisfy the following condition:

$$\frac{\alpha_M - \beta_E}{\kappa(\alpha_M - \alpha_m)} = \frac{\alpha_M - \beta_C}{\alpha_M - \alpha_m} = \eta. \tag{16}$$

(The general condition $\alpha_m \leq \beta_E \leq \beta_C \leq \alpha_M$ must hold in all cases for an equilibrium to be feasible.)

The equilibrium values in this general case can be expressed as follows:

$$\begin{cases} x_{C,e} = & \frac{\gamma}{\delta}\left(\lambda - \eta \frac{s}{\delta}(1+\kappa\psi)\right) \\ x_{E,e} = & \psi x_{C,e} \\ y_e = & \eta \frac{s}{\delta}(1+\kappa\psi) \\ w_e = & \eta\rho(1+\kappa\psi)x_{C,e} \end{cases}. \tag{17}$$

The free parameter, ψ, is the equilibrium ratio $x_{E,e}/x_{C,e}$, apparent from the second equation in Eq. (17). As opposed to φ, ψ cannot be easily related to the initial conditions; rather, it can be determined from the result of a simulation.

Again, the total population $x_e = x_{C,e} + x_{E,e}$ can be maximized by choosing δ appropriately:

$$\delta_{***} = \frac{2\eta s}{\lambda}(1+\kappa\psi). \tag{18}$$

This required depletion rate δ_{***} can be even larger than the optimal δ given by Eq. (14) depending upon the values of κ and ψ. In the presence of inequality, the maximum total population is no longer independent of κ and ψ and is smaller than the maximum carrying capacity given by Eqs. (11) and (15):

$$x_{e,M} = (1+\psi)\frac{\gamma}{\delta_{***}}\frac{\lambda}{2} = \frac{\gamma}{\eta s}\left(\frac{\lambda}{2}\right)^2\left(\frac{1+\psi}{1+\kappa\psi}\right). \tag{19}$$

5. Scenarios

We discuss three sets of scenarios:

1. Egalitarian society (No-Elites): Scenarios in which $x_E = 0$.
2. Equitable society (with Workers and Non-Workers): Scenarios in which $x_E \geq 0$ but $\kappa \equiv 1$.
3. Unequal society (with Elites and Commoners): Scenarios in which $x_E \geq 0$ and $\kappa > 1$.

For all of these scenarios, we start the model with the typical parameter values and initial conditions given in Table 1, unless otherwise stated. As indicated above, the values of κ and $x_E(0)$ determine the type of the society. Within each type of society, we obtain different scenarios by varying the depletion factor, δ.

In this section, we will show that HANDY is capable of modeling three distinct types of societies by changing κ and $x_E(0)$. A sustainable equilibrium can be found for each society by controlling δ. An appropriate choice of δ can make this equilibrium optimal, i.e., with maximum total population. Increasing δ above its optimal value makes the approach toward equilibrium oscillatory. Such an equilibrium is suboptimal, and the Carrying Capacity is below its maximum value, χ_M. It is also possible to reach a suboptimal equilibrium (a less than maximum, but sustainable population) by making δ *lower* than its optimal value.

Table 1
Description of parameters and state variables used in HANDY. κ, δ, and x_E are varied to study various scenarios in three different types of societies. $x_E = 0$ defines an Egalitarian society with no Elites. $\kappa = 1$ defines an Equitable society with Workers and Non-Workers, represented by x_C and x_E in this case, respectively. $x_E \geq 0$ and $\kappa > 1$ define an unequal society with Elites and Commoners (x_E and x_C). As a reference, all other variables and functions in HANDY are also listed above. Subscript e denotes *equilibrium* value everywhere in this paper.

Parameter symbol	Parameter name	Typical value(s)
α_m	Normal (minimum) death rate	1.0×10^{-2}
α_M	Famine (maximum) death rate	7.0×10^{-2}
β_C	Commoner birth rate	3.0×10^{-2}
β_E	Elite birth rate	3.0×10^{-2}
s	Subsistence salary per capita	5.0×10^{-4}
ρ	Threshold wealth per capita	5.0×10^{-3}
γ	Regeneration rate of nature	1.0×10^{-2}
λ	Nature carrying capacity	$1.0 \times 10^{+2}$
κ	Inequality factor	1, 10, 100
δ	Depletion (production) factor	None
Variable symbol	**Variable name**	**Typical initial value(s)**
x_C	Commoner population	$1.0 \times 10^{+2}$
x_E	Elite population	0, 1, 25
y	Nature	λ
w	Accumulated wealth	0

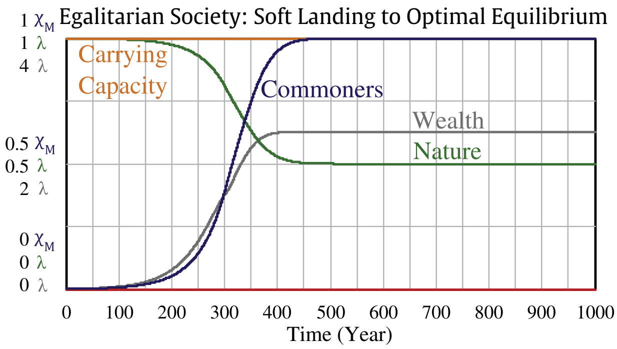
(a). List of parameters in HANDY. κ and δ take different values for different scenarios.
(b). List of state variables in HANDY. $x_E(0)$ takes different values for different scenarios.

However, in the latter case, the approach toward equilibrium would be a soft landing rather than oscillatory. When δ is increased even further, the society goes into cycles of prosperity and collapse. Increasing δ beyond a certain point will result in an irreversible Type-N (full) collapse, examples of which are presented in Sections 5.1.4, 5.2.4, and 5.3.2. We give a full categorization of collapses in the next two paragraphs.

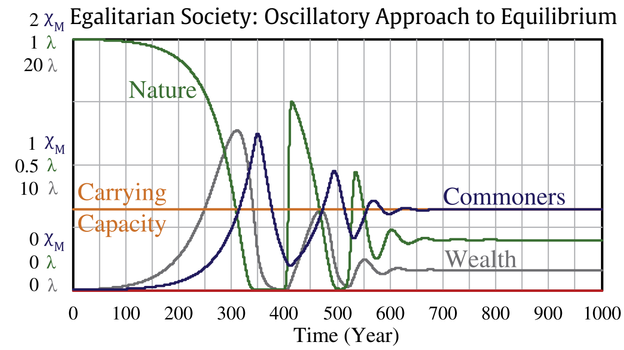
Running the model in different scenarios produces two kinds of collapses, either due to scarcity of labor (following an *inequality-induced* famine) or due to scarcity of Nature (depletion of natural resources). We categorize the former case as a Type-L (Disappearance of Labor) Collapse and the latter as a Type-N collapse (Exhaustion of Nature). In a Type-L collapse, growth of the Elite Population strains availability of resources for the Commoners. This causes decline of the Commoner Population (which does the labor), and consequently, decline of Wealth. Finally, Elite Population plummets since its source of subsistence, i.e., Wealth, has vanished. See Fig. 6a for an example of a Type-L collapse. This could represent a historical case such as the disappearance of the Mayan civilization in the Yucatan. Note that this type of collapse can only happen in an unequal society, because the major cause behind it is *inequality*.

Table 2
As a reference, all other variables and functions in HANDY are listed in this table. Subscript e denotes *equilibrium* value everywhere in this paper.

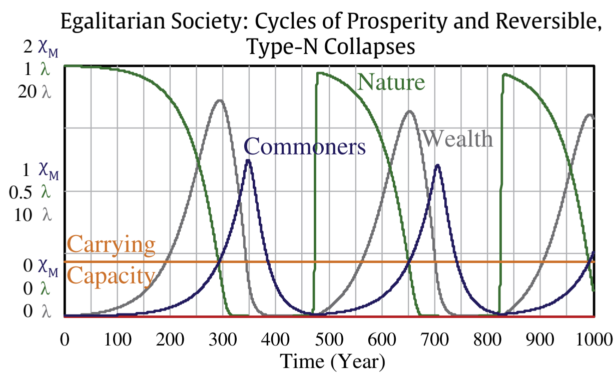
Variable symbol	Variable name	Defining equation
w_{th}	Threshold wealth	Eq. (5)
ω	Normalized wealth	w / w_{th}
C_C	Commoner consumption	Eq. (4) (Fig. 2a)
C_E	Elite consumption	Eq. (4) (Fig. 2a)
α_C	Commoner death rate	Eq. (6) (Fig. 2b)
α_E	Elite death rate	Eq. (6) (Fig. 2b)
η	η	Eq. (7)
χ	Carrying Capacity (C_C)	Eq. (9)
δ_*	Egalitarian optimal δ	Eq. (10)
χ_M	Maximum Carrying Capacity (Max C_C)	Eq. (11)
φ	Ratio of non-workers to workers (Equitable)	Eq. (12)
δ_{**}	Equitable optimal δ	Eq. (14)
ψ	Elite to commoner equilibrium ratio (Unequal)	$x_{E,e} / x_{C,e}$
δ_{***}	Unequal optimal δ	Eq. (18)



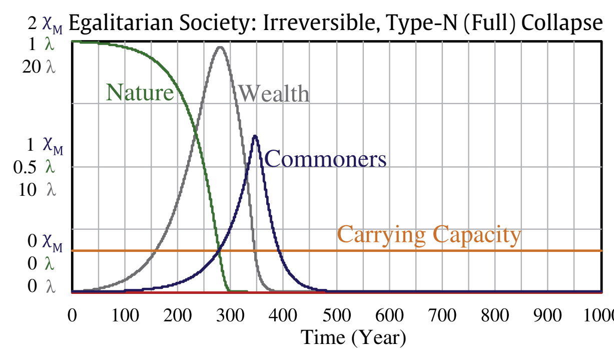
a) Soft landing to the optimal equilibrium when Elite population (marked in red) equals zero. Final population reaches the carrying capacity, which is at its maximum value, χ_M, in this scenario.



b) Oscillatory approach to equilibrium when Elite population (marked in red) equals zero. Final population converges to the carrying capacity, which is lower than its maximum value, χ_M, in this scenario.



c) Cycles of prosperity, overshoot, (reversible Type-N) collapse, and revival when Elite population (marked in red) equals zero.



d) Irreversible Type-N collapse (full collapse) when Elite population (marked in red) equals zero. All the state variables collapse to *zero* in this scenario due to over-depletion.

Fig. 3. Experiment results for the Egalitarian society.

A Type-N collapse, on the other hand, starts with an exhaustion of Nature, followed by a decline of Wealth that in turn, causes a fall of the Commoners and then the Elites. Depending on the depletion rate, Type-N collapses can be "reversible" or "irreversible". After a reversible collapse, regrowth of nature can trigger another cycle of prosperity, examples of which can be seen in Figs. 3c and 4c. This could represent historical cases such as the Greek and Roman collapses.

When depletion is pushed beyond a certain limit, Nature fully collapses and the whole system completely collapses after that. This is why we call an irreversible Type-N collapse a "full" collapse. Examples of such collapses can be seen in Figs. 3d, 4d, and 6b. This could represent a historical case such as the exhaustion of Nature on Easter Island. Type-N collapses can arise because of excessive depletion *only* (Figs. 3d and 4d), or both excessive depletion and *inequality* (Fig. 6b).

It is important to understand the inter-relation of the depletion factor, δ, and the Carrying Capacity, χ. The further δ is taken away from its optimal value, the further χ moves down from its maximum value, χ_M. An equilibrium can be reached if and only if χ is not too far away from χ_M, which means δ cannot be too far away from its optimal value, given by Eqs. (10), (14), and (18) in the three types of societies under consideration. Note that in all of the scenario outputs presented below (for the three types of societies under consideration), Carrying Capacity (χ) and the Maximum Carrying Capacity (χ_M) are calculated from their defining Eqs. (9) and (11), respectively.

Important note about the units of the vertical axis of all the subsequent graphs: Populations, x_C and x_E, and the Carrying Capacity, χ, are all normalized to the Maximum Carrying Capacity, χ_M. Nature and Wealth are both shown in units of Nature's capacity, λ. The top scale of the vertical axis of the graph pertains to Population(s) and Carrying Capacity; the middle scale pertains to Nature, which (normally) stays bounded by 1λ; and the bottom scale is for Wealth.

Note: All the simulations below use the Euler integration method with a time-step of 1 year and single precision.

5.1. Egalitarian Society (No-Elites): $x_E = 0$

In the four following scenarios, κ does not play any role since we set $x_E \equiv 0$. We start the depletion rate from $\delta = \delta_*$, the optimal equilibrium value that maximizes the Carrying Capacity, and increase it slowly to get additional scenarios. The horizontal red line in the graphs for the four scenarios of this section represents the *zero* population of Elites.

5.1.1. Egalitarian Society: Soft Landing to Equilibrium

For the scenario in Fig. 3a, $\delta = \delta_* = 6.67 \times 10^{-6}$. Therefore, the carrying capacity, χ, is at its maximum level, χ_M. Notice that Nature also settles to $y_e = \lambda / 2$, which is the value that results in the maximum regeneration rate. This maximal regeneration can in turn support a maximum sustainable depletion and population.

If we set $\delta < \delta_*$, we still see a soft landing to the carrying capacity, χ. However, χ would be at a lower level than χ_M because a lower-than-optimal δ does not correspond to the maximum regeneration of nature, which is a necessity if we want to have the *maximum sustainable* population. The advantage of a lower-than-optimal δ is a higher equilibrium level (compared to λ / 2) for Nature.

Choosing a depletion rate, δ, that is too small to produce enough to feed the population would result in a collapse, and thus make any equilibrium impossible even though Nature stays at its maximum capacity. Of course, this would not occur in the real world as the urge for survival guarantees humans extract their basic needs from nature.

5.1.2. Egalitarian Society: Oscillatory Approach to Equilibrium

For the scenario in Fig. 3b, δ is increased to $\delta = 2.5\delta_* = 1.67 \times 10^{-5}$. As can be seen from Fig. 3b, the carrying capacity, χ, is lower than its maximum value, χ_M. Population initially overshoots the carrying capacity, then oscillates, and eventually converges to it since the amount of overshoot is not too large, just about the order of χ. Note that at the time the (total) population overshoots the Carrying Capacity, the Wealth also reaches a maximum and starts to decline.

5.1.3. Egalitarian Society: Cycles of Prosperity, Overshoot, Collapse, and Revival

For the scenario in Fig. 3c, δ is increased to $\delta = 4\delta_* = 2.67 \times 10^{-5}$. As can be seen, Population, Nature and Wealth all collapse to a very small value. However, after depletion becomes small due to very low number of workers, Nature gets a chance to grow back close to its capacity, λ. The regrowth of Nature kicks off another cycle of prosperity which ends with another collapse. Simulation results show that these cycles, ending in Type-N collapses (i.e., those that start due to scarcity of Nature), repeat themselves indefinitely. Therefore, such cycles represent "reversible" Type-N collapses. This reversibility is possible as long as δ stays within a "safe" neighborhood of δ_*.

5.1.4. Egalitarian Society: Irreversible Type-N Collapse (Full Collapse)

For the scenario in Fig. 3d, δ is increased further to $\delta = 5.5\delta_* = 3.67E - 5$. The overshoot is so large that it forces Population, Nature and Wealth into a full collapse, after which there is no recovery. This is a generic type of collapse that can happen for any type of society due to over-depletion. See Sections 5.2.4 and 5.3.2 for examples of irreversible Type-N collapses in equitable and unequal societies, respectively. We include further discussion of these two types of collapses in Section 6.

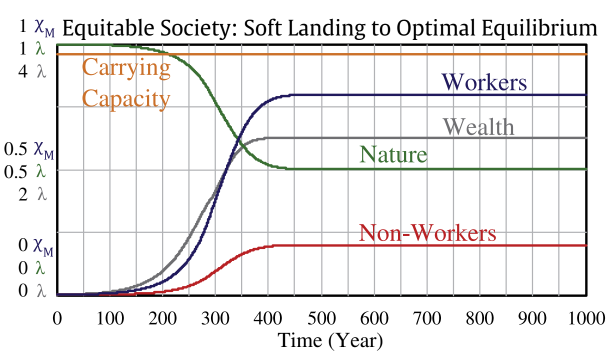
We observe that the accumulated Wealth delays a decline of the population even after Nature has declined well below its capacity, λ. Therefore, population keeps growing and depleting Nature until Nature is fully exhausted. At that instance, i.e., when $y = 0$, Wealth cannot grow any further; indeed, it starts plummeting, causing a sharp fall of the population level, and eventually its full, irreversible collapse.

5.2. Equitable Society (with Workers and Non-Workers): $\kappa = 1$

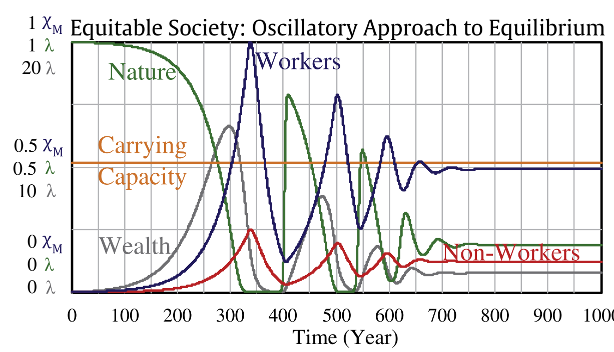
We take the parameter values and the initial conditions to be the same as in Table 1, except that this time we set $x_E(0) = 25$ ($\varphi = 0.25$) and $\kappa = 1$. We start with the optimal depletion per capita $\delta = \delta_{**}$, which will sustain the maximum population (see Eq. (14)), and will gradually increase it in order to get the additional scenarios in this subsection. Notice that in these cases, x_C describes the Working Population, while x_E describes the Non-Working Population. Everybody consumes at the same level, since we set $\kappa = 1$, i.e., we assume there is no inequality in consumption level for Workers and Non-Workers.

5.2.1. Equitable Society: Soft Landing to Optimal Equilibrium

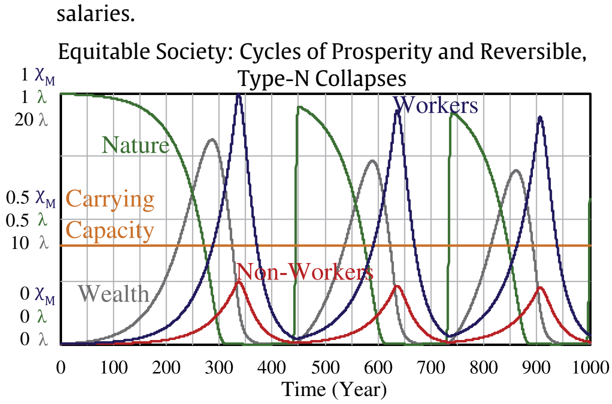
For the scenario in Fig. 4a, $\delta = \delta_{**} = 8.33 \times 10^{-6}$. Notice that this is larger than the optimal value in the absence of Non-Workers $\delta_* =$



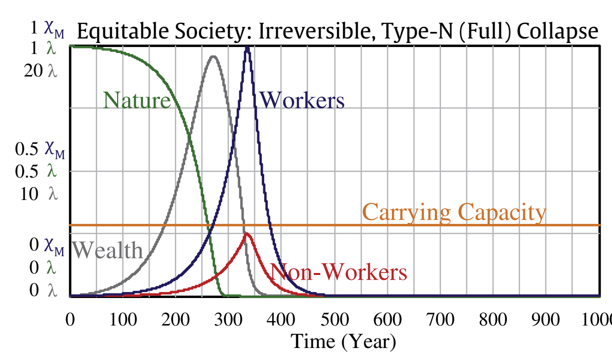
a) Equilibrium in the presence of both Workers and Non-Workers can be attained with slow growth and equitable salaries.



b) Oscillatory approach to equilibrium in the presence of both Workers and Non-Workers is possible when the overshoot is not too large.



c) Cycles of prosperity, overshoot, (reversible Type-N) collapse, and revival in the presence of Workers and Non-Workers.



d) Irreversible Type-N collapse (full collapse) happens after a period of very fast growth.

Fig. 4. Experiment results for the Equitable society.

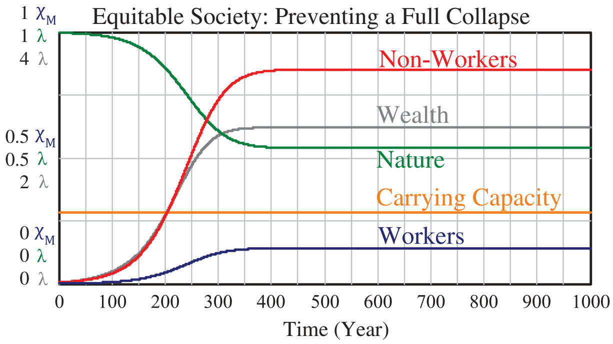


Fig. 5. The full collapse that happened in the previous scenario, Fig. 4d, can be prevented by reducing the average depletion per capita. This can be achieved by either increasing the ratio of the Non-Working to Working population (high δ, high φ) or decreasing the average workload per worker, i.e., decreasing total work hours per week (low δ, low φ).

6.67×10^{-6} even though all the other parameters are identical to those in Section 5.1.1. This difference arises because $x_E \neq 0$, which in turn forces the Workers to produce extra in order to support the Non-Workers. Now, $\chi < \chi_M$ because $\delta = \delta_{**} \neq \delta_*$. However, by setting $\delta = \delta_{**}$, the optimal value of δ in the presence of Non-Workers, the total population, $x_C + x_E$ still reaches the maximum Carrying Capacity, χ_M, the same as in Section 5.1. See Eq. (15) and Section 4.2 for a mathematical description.

Similar comments as in Section 5.1.1 apply here when we choose a lower-than-optimal δ.

5.2.2. Equitable Society: Oscillatory Approach to Equilibrium

For the scenario in Fig. 4b, $\delta = 2.64\delta_{**} = 2.20 \times 10^{-5}$. The total population is equal to the actual Carrying Capacity (smaller than the maximum Carrying Capacity).

5.2.3. Equitable Society: Cycles of Prosperity, Overshoot, Collapse, and Revival

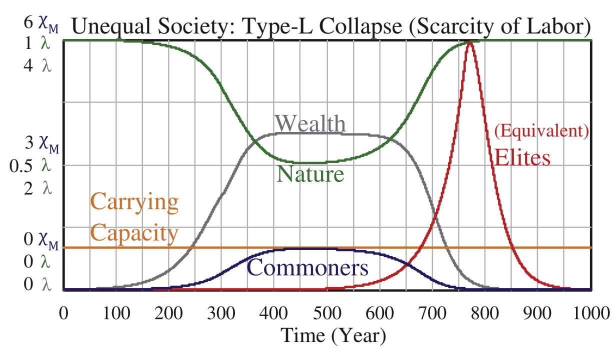
For the scenario in Fig. 4c, $\delta = 3.46\delta_{**} = 3.00 \times 10^{-5}$. The result is analogous to Fig. 3c which corresponds to Section 5.1.3. As before, the time at which the total population overshoots the actual Carrying Capacity is indicated by the fact that Wealth starts to decrease. After each cycle of prosperity, there is a partial, reversible Type-N collapse.

5.2.4. Equitable Society: Full Collapse

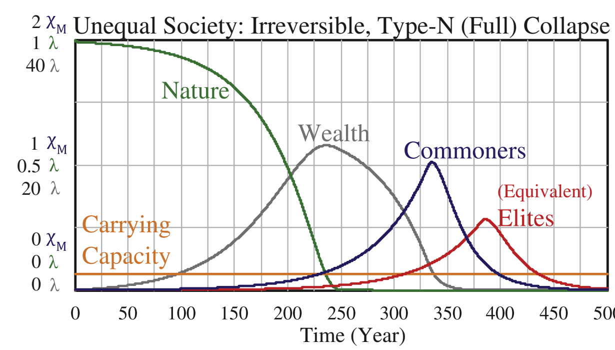
For the scenario in Fig. 4d, $\delta = 5\delta_{**} = 4.33 \times 10^{-5}$. Once again, we can see how an irreversible Type-N (full) collapse of Population, Nature, and Wealth can occur due to over-depletion of natural resources as a result of high depletion per capita.

5.2.5. Equitable Society: Preventing a Full Collapse by Decreasing Average Depletion per Capita

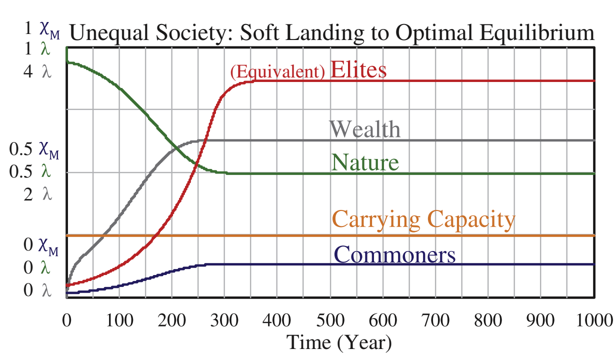
The case in Fig. 5 is similar to the previous case (see Section 5.2.4 and Fig. 4d), except that we raised the ratio of Non-Workers to Workers, φ, from 0.25 to 6. This corresponds to changing $x_E(0)$ from 25 to 600,



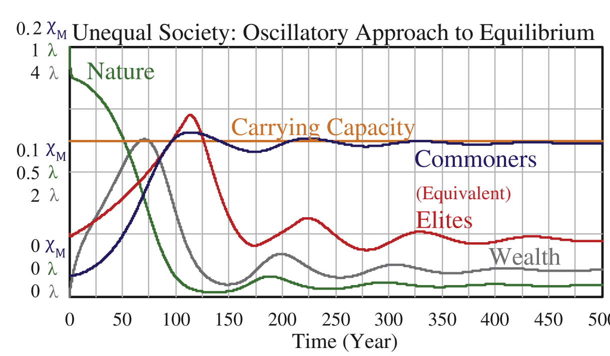
a) Population collapse following an apparent equilibrium due to a small initial Elite population when $\kappa = 100$. This scenario also shows a different route to a collapse, in which, although Nature eventually recovers, population does not.



b) A fast full collapse due to both over-depletion and inequality ($\kappa = 100$).



c) With moderate in equality ($\kappa = 10$), it is possible to attain an optimal equilibrium by controlling the birth rates.



d) With $\delta \gtrsim \delta_{***}$, it is still possible to oscillate and converge to an equilibrium ($\kappa = 10$).

Fig. 6. Experiment results for the Unequal society.

while keeping $x_C(0) = 100$. By increasing the ratio of non-workers to workers, a sustainable equilibrium can be reached due to lower average depletion per capita — an equivalent δ if everyone contributed equally to labor. This could also be interpreted as modeling a reduction in the average workload per worker.

5.3. Unequal Society (with Elites and Commoners): $x_E \geq 0$ and $\kappa > 1$

In our examples of an unequal society, the Elites (per capita) consume $\kappa <$ 10 to 100 times more than the Commoners. Their population, plotted in red, is multiplied by κ to represent their equivalent impact because of their higher consumption. That is why we use the label "*Equivalent* Elites" on the graphs in this Section 5.3.

In the first two cases, we discuss two distinct, but generic types of collapse in an unequal society. In these two scenarios, $\kappa = 100$. Then we will show possibility of reaching an equilibrium by reducing κ to 10 and adjusting the birth rates β_E and β_C independently. These two $\kappa = 10$ scenarios show that in order to reach a sustainable equilibrium in an unequal society, it is necessary to have policies that limit inequality and ensure birth rates remain below critical levels.

5.3.1. Unequal Society: Type-L Collapse (Labor Disappears, Nature Recovers)

This scenario, presented in Fig. 6a, is precisely the same as the equilibrium without Elites case presented in Section 5.1.1 (Fig. 3a) except that here we set $x_E(0) = 1.0 \times 10^{-3}$. This is indeed a very small initial seed of Elites. The two scenarios look pretty much the same up until about $t = 500$ years after the starting time of the simulation. The Elite population starts growing significantly only after $t = 500$, hence depleting the Wealth and causing the system to collapse. Under this scenario, the system collapses due to worker scarcity even though natural resources are still abundant, but because the depletion rate is optimal, it takes more than 400 years after the Wealth reaches a maximum for the society to collapse. In this example, Commoners die out first and Elites disappear later. This scenario shows that in a society that is otherwise sustainable, the highly unequal consumption of elites will still cause a collapse.

This scenario is an example of a Type-L collapse in which both Population and Wealth collapse but Nature recovers (to its maximum capacity, λ, in the absence of depletion). Scarcity of workers is the initial cause of a Type-L collapse, as opposed to scarcity of Nature for a Type-N collapse.

5.3.2. Unequal Society: Irreversible Type-N Collapse (Full Collapse)

The typical scenario in Fig. 6b for a full collapse is the result of running the model with the parameter values and initial conditions given by Table 1. Examples of irreversible Type-N (full) collapses in the egalitarian and equitable societies are presented in Section 5.1.4 (Fig. 3d) and Section 5.2.4 (Fig. 4d).

We set a small initial seed of $x_E(0) = 0.20$, $\kappa = 100$, and a large depletion $\delta = 1.0 \times 10^{-4}$, so that both the depletion $\delta = 15\delta_*$ and the inequality coefficient $\kappa = 100$ are very large. This combination results in a full collapse of the system with no recovery. The Wealth starts declining as soon as the Commoner's population goes beyond its carrying capacity, and then the full collapse takes only about 250 additional years. The declining Wealth causes the fall of the Commoner's population (workers) with a time lag. The fast reduction in the number of workers combined with scarcity of natural resources causes the Wealth to decline even faster than before. As a result, the Elites – who could initially survive the famine due to their unequal access to consumable goods ($\kappa = 100$) – eventually also die of hunger. Note that because both depletion and inequality are large, the collapse takes place faster and at a much lower level of population than in the previous case (see Section 5.3.1, Fig. 5.3.1) with a depletion rate of $\delta = \delta_*$.

5.3.3. Unequal Society: Soft Landing to Optimal Equilibrium

The following parameter values and initial values can produce the current scenario (the rest are exactly the same as in Table 1):

$$\begin{cases} \beta_C &= 6.5 \times 10^{-2} & \beta_E &= 2.0 \times 10^{-2} \\ x_C(0) &= 1.0 \times 10^{+4} & x_E(0) &= 3.0 \times 10^{+3} \\ \kappa &= 10 & \delta &= 6.35 \times 10^{-6} \end{cases}. \tag{20}$$

The value for δ used in this scenario is δ_{***} given by Eq. (18). It must be remembered that $\psi = 0.65$ is not a parameter that we can choose. However, it can be read from the result of the simulation since it is the equilibrium ratio of the Elite to Commoner population. See the second equation in Eq. (17). On the other hand, $\eta = \frac{1}{12}$ is determined by the death and birth rates as well as the inequality coefficient. These parameters are chosen in order to satisfy Eq. (16), the necessary condition for attaining an equilibrium in an unequal society.

The same comments as in Section 5.1.1 hold here if we choose a lower-than-optimal δ.

5.3.4. Unequal Society: Oscillatory Approach to Equilibrium

The parameter values and initial conditions in the scenario presented in Fig. 6d are exactly the same as the previous scenario, presented in Fig. 6c, except for δ. It is increased to 1.3×10^{-5}, almost $2\delta_{***}$. This results in a much lower Carrying Capacity compared to 5.3.3, as can be seen from a comparison of Fig. 6c and d. Therefore, the total final population in the present scenario is much less than the total final population in the previous scenario, 5.3.3 (Fig. 6c) (Table 2).

6. Discussion of Results

We conducted a series of experiments with the HANDY model, considering first an egalitarian society without Elites ($x_E = 0$), next an equitable society ($\kappa = 1$) where Non-Workers and Workers are equally paid, and finally an unequal society whose Elites consume κ times more than the Commoners. The model was also used to find a sustainable equilibrium value and the maximum carrying capacity within each of these three types of societies.

6.1. Unequal Society

The scenarios most closely reflecting the reality of our world today are found in the third group of experiments (see the scenarios for an unequal society in Section 5.3), where we introduced economic stratification. Under such conditions, we find that collapse is difficult to avoid, which helps to explain why economic stratification is one of the elements recurrently found in past collapsed societies. Importantly, in the first of these unequal society scenarios, 5.3.1, *the solution appears to be on a sustainable path for quite a long time*, but even using an optimal depletion rate (δ_*) and starting with a very small number of Elites, the Elites eventually consume too much, resulting in a famine among Commoners that eventually causes the collapse of society. It is important to note that this Type-L collapse is *due to an inequality-induced famine that causes a loss of workers*, *rather than a collapse of Nature*. Despite appearing initially to be the same as the sustainable optimal solution obtained in the absence of Elites, economic stratification changes the final result: Elites' consumption keeps growing until the society collapses. The Mayan collapse – in which population never recovered even though nature *did* recover – is an example of a Type-L collapse, whereas the collapses in the Easter Island and the Fertile Crescent – where nature was depleted – are examples of a Type-N collapse.

In scenario 5.3.2, with a larger depletion rate, the decline of the Commoners occurs faster, while the Elites are still thriving, but eventually the Commoners collapse completely, followed by the Elites. It is important to note that in both of these scenarios, the Elites – due to their wealth – do not suffer the detrimental effects of the environmental collapse until much later than the Commoners. This buffer of wealth allows Elites to continue "business as usual" despite the impending catastrophe. It is likely that this is an important mechanism that would help explain how historical collapses were allowed to occur by elites who appear to be oblivious to the catastrophic trajectory (most clearly apparent in the Roman and Mayan cases). This buffer effect is further reinforced by the long, apparently sustainable trajectory prior to the beginning of the collapse. While some members of society might raise the alarm that the system is moving towards an impending collapse and therefore advocate structural changes to society in order to avoid it, Elites and their supporters, who opposed making these changes, could point to the long sustainable trajectory "so far" in support of doing nothing.

The final two scenarios in this set of experiments, 5.3.3 and 5.3.4, are designed to indicate the kinds of policies needed to avoid this catastrophic outcome. They show that, in the context of economic stratification, inequality must be greatly reduced and population growth must be maintained below critical levels in order to avoid a societal collapse (Daly, 2008).

6.2. Egalitarian Society

In order to further understand what conditions are needed to avoid collapse, our first set of experiments model a society without economic stratification and start with parameter values that make it possible to reach a maximum carrying capacity (scenario 5.1.1). The results show that in the absence of Elites, if the depletion per capita is kept at the optimal level of δ_*, the population grows smoothly and asymptotes the level of the maximum carrying capacity. This produces a soft-landing to equilibrium at the maximum sustainable population and production levels.

Increasing the depletion factor slightly (scenario 5.1.2) causes the system to oscillate, but still reach a sustainable equilibrium, although, importantly, at a lower carrying capacity. Population overshoots its carrying capacity, but since the overshoot is not by too much – of the order of the carrying capacity – the population experiences smaller collapses that can cause it to oscillate and eventually converge to a sustainable equilibrium. Thus, while social disruption and deaths would occur, a total collapse is avoided.

A further increase in the depletion factor (scenario 5.1.3) makes the system experience oscillatory periods of growth, very large overshoots and devastating collapses that almost wipe out society, but the eventual recovery of Nature allows for the cycle to be repeated.

Increasing the depletion factor even further (scenario 5.1.4) results in a complete collapse of the system. This shows that depletion alone, if large enough, can result in a collapse — even in the absence of economic stratification.

6.3. Equitable Society (with Workers and Non-Workers)

As the second set of experiments (presented in Section 5.2) show, HANDY allows us to model a diverse range of societal arrangements. In this set of experiments, choosing $x_E \geq 0$ and $\kappa = 1$ has allowed us to model a situation that can be described as having Workers and Non-Workers with the same level of consumption, i.e., with no economic stratification. The Non-Workers in these scenarios could represent a range of societal roles from students, retirees, and disabled people, to intellectuals, managers, and other non-productive sectors. In this case, the Workers have to deplete enough of Nature to support both the Non-Workers and themselves.

The first scenario, 5.2.1, shows that even with a population of Non-Workers, the total population can still reach a sustainable equilibrium without a collapse. In scenario 5.2.2, we find that increasing the depletion factor induces a series of overshoots and small collapses where population eventually converges to a lower sustainable equilibrium. Like in an egalitarian society, scenario 5.2.3 shows us that increasing the depletion parameter further results in cycles of large overshooting, major collapses, and then eventual recovery of Nature. Scenario 5.2.4 shows us that increasing depletion per capita further can produce an irreversible Type-N collapse.

Finally, scenario 5.2.5, which is a replication of the scenario in 5.2.4 with a much higher ratio of Non-Workers to Workers, shows that a collapse in an equitable society could be avoided by reducing the average depletion per capita. We note that this scenario could also represent a situation where, rather than having paid Non-Workers, the workload per capita is reduced, with the whole population working "fewer days a week". Such a "work-sharing" policy has been successfully implemented in Germany over the past few years for reducing unemployment (Baker and Hasset, 2012; Hasset, 2009). Moreover, Knight et al. (2013) show, through a panel analysis of data for 29 high-income OECD countries from 1970 to 2010, that reducing work hours can contribute to sustainability by reducing ecological strain. This conclusion agrees with our comparison of the two scenarios, 5.2.5 and 5.2.4, presented above.

6.4. HANDY and Brander–Taylor Model

As previously mentioned, a similar use of the predator–prey approach was applied in the pioneering work of Brander and Taylor (1998) (BT) to study the historical rise and fall of the Easter Island population. In comparison to their model, with just two equations for Population and Nature, the introduction of Elites and Commoners, and accumulated Wealth, results in a greater variety and broader spectrum of potential solutions. Moreover, the collapse scenario presented in BT is somewhat different from the ones presented above. As a matter of fact, the collapse scenario presented in Fig. 3 of Brander and Taylor (1998) seems to be more of an oscillatory approach to equilibrium, similar to the one shown in our Fig. 3b, and not a collapse in the sense that we define in this paper. Furthermore, the carrying capacity, in the sense we define in this paper, is also different from what BT (1998) call carrying capacity. Indeed, their carrying capacity (K) is our Nature's capacity, λ, which is the maximum size Nature can reach, whereas Carrying Capacity in HANDY is the population level that can be supported by a given level of natural resources. Furthermore, BT's carrying capacity is a constant, whereas Carrying Capacity in HANDY adjusts according to the level of depletion of Nature.

While sharing certain similarities with the Brander and Taylor model, our more complex model structure and the use of different assumptions, allows our model to apply to multiple types of societies with varying socioeconomic structures. Thus, unlike works that tend to study further implications of the two-dimensional model of BT (Anderies, 2000), the model we have developed introduces a more complex set of possible feedbacks and nonlinear dynamics, and a greater spectrum of potential outcomes. This allows HANDY to model a different and wider set of thought experiments.

An important feature of HANDY that distinguishes it from predator–prey, BT, and other similar models (Anderies, 1998; Dalton et al., 2005; Erickson and Gowdy, 2000; Reuveny and Decker, 2000) is its native capability for producing irreversible collapses due to the structure for accumulation of wealth. Our approach also differs from models like D'Alessandro (2007) that can produce irreversible collapses but only through explicit introduction of a critical depensation mechanism into the model. The dynamics produced by HANDY offer the possibility of irreversible collapses without having to introduce such an additional mechanism into the model. See Section 5.1.4 for an explanation of irreversible collapses in HANDY.[1]

[1] We wish to acknowledge and thank reviewer No. 1 for highlighting these very important points to us.

7. Summary

Collapses of even advanced civilizations have occurred many times in the past five thousand years, and they were frequently followed by centuries of population and cultural decline and economic regression. Although many different causes have been offered to explain individual collapses, it is still necessary to develop a more general explanation. In this paper we attempt to build a simple mathematical model to explore the essential dynamics of interaction between population and natural resources. It allows for the two features that seem to appear across societies that have collapsed: the stretching of resources due to strain placed on the ecological carrying capacity, and the division of society into Elites (rich) and Commoners (poor).

The Human And Nature DYnamical model (HANDY) was inspired by the predator and prey model, with the human population acting as the predator and nature being the prey. When small, Nature grows exponentially with a regeneration coefficient γ, but it saturates at a maximum value λ. As a result, the maximum regeneration of nature takes place at $\lambda / 2$, not at the saturation level λ. The Commoners produce wealth at a per capita depletion rate δ, and the depletion is also proportional to the amount of nature available. This production is saved as accumulated wealth, which is used by the Elites to pay the Commoners a subsistence salary, s, and pay themselves κs, where κ is the inequality coefficient. The populations of Elites and Commoners grow with a birth rate β and die with a death rate α which remains at a healthy low level when there is enough accumulated food (wealth). However, when the population increases and the wealth declines, the death rate increases up to a famine level, leading to population decline.

We show how the carrying capacity – the population that can be indefinitely supported by a given environment (Catton, 1980) – can be defined within HANDY, as the population whose total consumption is at a level that equals what nature can regenerate. Since the regrowth of Nature is maximum when $y = \lambda / 2$, we can find the optimal level of depletion (production) per capita, δ_* in an egalitarian society where $x_E \equiv 0$, $\delta_{**}(\geq \delta_*)$ in an equitable society where $\kappa \equiv 1$, and δ_{***} in an unequal society where $x_E \geq 0$ and $\kappa > 1$.

In sum, the results of our experiments, discussed in Section 6, indicate that either one of the two features apparent in historical societal collapses – over-exploitation of natural resources and strong economic stratification – can independently result in a complete collapse. Given economic stratification, collapse is very difficult to avoid and requires major policy changes, including major reductions in inequality and population growth rates. Even in the absence of economic stratification, collapse can still occur if depletion per capita is too high. However, collapse can be avoided and population can reach equilibrium if the per capita rate of depletion of nature is reduced to a sustainable level, and if resources are distributed in a reasonably equitable fashion.

In the upcoming generations of HANDY, we plan to develop several extensions including: (1) disaggregation of Nature into nonrenewable stocks, regenerating stocks, and renewable flows, as well as the introduction of an investment mechanism in accessibility of natural resources, in order to study the effects of investment in technology on resource choice and production efficiency; (2) making inequality (κ) endogenous to the model structure; (3) introduction of “policies” that can modify parameters such as depletion, the coefficient of inequality, and the birth rate; and, (4) introduction of multiple coupled regions to represent countries with different policies, trade of carrying capacity, and resource wars.

Those interested in obtaining the model code can contact the authors.

Acknowledgments

We are grateful to Profs. Matthias Ruth, Victor Yakovenko, Herman Daly, Takemasa Miyoshi, Jim Carton, Fernando Miralles-Wilhelm, and Ning Zeng, and Drs. Robert Cahalan and Steve Penny for many useful discussions. Study of the “Equitable Society” scenarios (i.e., with Workers and Non-Workers), the scenario presented in Section 5.2.5, in particular, was suggested by V. Yakovenko. We would also like to thank anonymous reviewer No. 1 for having highlighted to us the importance of the capability of HANDY to naturally produce irreversible collapses, which is not found in earlier models. We would especially like to thank the editors of this journal for alerting us to the model and work done by Brander and Taylor, of which we were unaware, and allowing us to revise our article to account for this new information. This work was partially funded through NASA/GSFC grant NNX12AD03A.

Based on the media reports on a pre-publication version of this paper, NASA issued the official statement contained in Release 14-082:

> http://www.nasa.gov/press/2014/march/nasa-statement-on-sustainability-study/
>
> March 20th, 2014
> RELEASE 14-082
> NASA Statement on Sustainability Study
>
> The following is a statement from NASA regarding erroneous media reports crediting the agency with an academic paper on population and societal impacts.
>
> "A soon-to-be published research paper 'Human and Nature Dynamics (HANDY): Modeling Inequality and Use of Resources in the Collapse or Sustainability of Societies' by University of Maryland researchers Safa Motesharrei and Eugenia Kalnay, and University of Minnesota’s Jorge Rivas was not solicited, directed or reviewed by NASA. It is an independent study by the university researchers utilizing research tools developed for a separate NASA activity."
>
> "As is the case with all independent research, the views and conclusions in the paper are those of the authors alone. NASA does not endorse the paper or its conclusions."

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